Calcul des déformations des fils élastiques

Fils élastiques en arc de cercle - Forces et couples concentrés

Déformations planes

Fil rond en acier

$d := 0.6 \cdot mm$ $\quad S := \pi \cdot \frac{d^2}{4}$ $\quad E := 2 \cdot 10^5 \cdot N \cdot mm^{-2}$ $\quad G := \frac{E}{2.6}$ $\quad \rho := 7.85 \cdot 10^3 \cdot kg \cdot m^{-3}$

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$J_t := J_{t\_circ}(d)$ $\quad I_{22} := I_{f\_circ}(d)$ $\quad I_{33} := I_{22}$

$W_t := W_{t\_circ}(d)$ $\quad W'_t := W_t$ $\quad W_{f2} := W_{f\_circ}(d)$ $\quad W_{f3} := W_{f2}$

Caractéristiques de l'arc de cercle $\quad R := 21 \cdot mm$ $\quad \psi_{AB} := 75 \cdot deg$

Forces extérieures en bout d'arc $\quad F := 0.2 \cdot N$ $\quad \psi_F := \psi_{AB}$ $\quad \lambda_F := 45 \cdot deg$

$F_x := F \cdot cos(\lambda_F)$ $\quad F_y := F \cdot sin(\lambda_F)$ $\quad F_z := 0 \cdot N$ $\quad C_x := 0 \cdot N \cdot mm$ $\quad C_y := 0 \cdot N \cdot mm$ $\quad C_z := 2 \cdot N \cdot mm$

$\mathbf{F} := (F_x \;\; F_y \;\; F_z)^T$ $\quad |\mathbf{F}| = 0.2\,N$ $\quad \mathbf{C} := (C_x \;\; C_y \;\; C_z)^T$ $\quad |\mathbf{C}| = 2\,N \cdot mm$

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Arc de cercle E_L - F&C.mcd(R)

Valeur de tests transitoires $\quad \alpha_m := 20 \cdot deg$

Torseur des forces de cohésion

$\mathbf{M_c}(\psi_F, \alpha_m)^T = (0 \;\; 0 \;\; -1.875)\,N \cdot mm$

Sollicitations

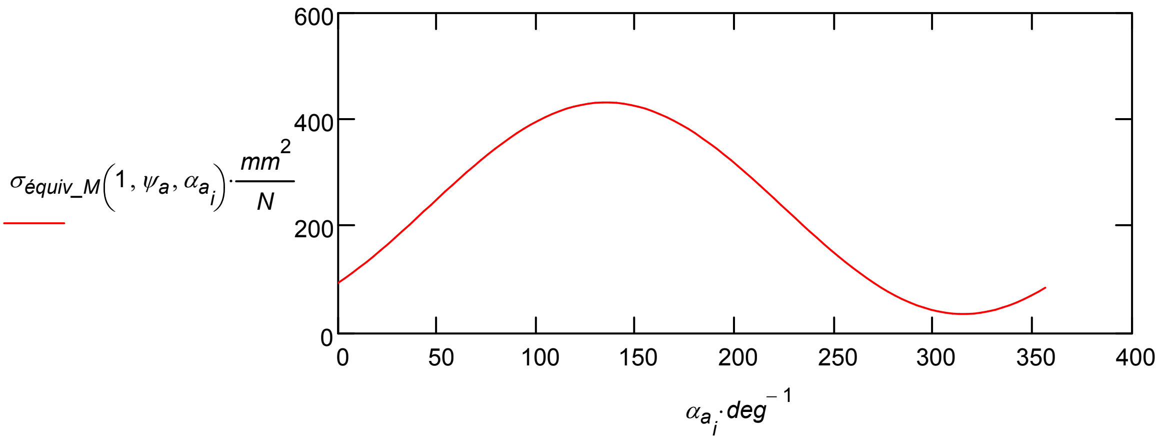
$\mathbf{e'_1}(\alpha_m)^T = (-0.342 \;\; 0.94 \;\; 0)$ $\quad \mathbf{e'_2}(\alpha_m)^T = (-0.94 \;\; -0.342 \;\; 0)$ $\quad \mathbf{e'_3}(\alpha_m)^T = (0 \;\; 0 \;\; 1)$

Moment de torsion $\quad M_t(\psi_F, \alpha_m) = 0\,N \cdot mm$

Moments de flexion $\quad M_{f2}(\psi_F, \alpha_m) = 0\,N \cdot mm$ $\quad M_{f3}(\psi_F, \alpha_m) = -1.875\,N \cdot mm$

Contraintes

Cas d'un anneau fendu $\quad n := 101$ $\quad i := 1 .. n-1$ $\quad \psi_a := 360 \cdot deg$ $\quad \alpha_{a_i} := (i-1) \cdot \frac{\psi_a}{n-1}$



Calcul des déplacements par les intégrales de Mohr

Position du déplacement désiré $\alpha_M := 40 \cdot deg$

Calcul des déplacements linéiques

Déplacement dans la direction de Ox $\lambda := 0 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_F, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \alpha_M, \lambda, \gamma) = 0.171\,mm$

$\delta_x(\psi_F, \alpha) := \delta_v(\psi_F, \alpha, \lambda, \gamma)$ $\boxed{\delta_x(\psi_F, \alpha_M) = 0.171\,mm}$

Déplacement dans la direction de Oy $\lambda := 90 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_F, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \alpha_M, \lambda, \gamma) = 0.081\,mm$

$\delta_y(\psi_F, \alpha) := \delta_v(\psi_F, \alpha, \lambda, \gamma)$ $\boxed{\delta_y(\psi_F, \alpha_M) = 0.081\,mm}$

Déplacement dans la direction de *R* $\lambda := \alpha_M$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_F, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \alpha_M, \lambda, \gamma) = 0.183\,mm$

$\delta_R(\psi_F, \alpha) := \delta_v(\psi_F, \alpha, \lambda, \gamma)$ $\boxed{\delta_R(\psi_F, \alpha_M) = 0.183\,mm}$

Calcul des déplacements angulaires

Déplacement angulaire autour de l'axe normal au plan de l'arc $\lambda_c := 0 \cdot deg$ $\gamma_c := 0 \cdot deg$ $|\mathbf{cv}(\lambda, \gamma)| = 1$

$\theta_{tcv}(\psi_F, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_{fcv2}(\psi_F, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_{fcv3}(\psi_F, \alpha_M, \lambda_c, \gamma_c) = -1.214\,deg$

$\theta_z(\psi_F, \alpha) := \theta_{fcv3}(\psi_F, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_z(\psi_F, \alpha_M) = -1.214\,deg}$

Solution analytique

Déplacements cartésiens en M par matrice de souplesse

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Arc de cercle E_L - Matrice S.mcd(R)

$\mathbf{T} := \left[F_x \cdot N^{-1} \quad F_y \cdot N^{-1} \quad C_z \cdot (N \cdot m)^{-1} \right]^T$ $\Delta(\psi_F, \alpha) := \mathbf{S_{PF}}(\psi_F, \alpha) \cdot \mathbf{T}$

$\delta_1(\psi_F, \alpha) := \Delta(\psi_F, \alpha)_1 \cdot m$ $\delta_2(\psi_F, \alpha) := \Delta(\psi_F, \alpha)_2 \cdot m$ $\theta_3(\psi_F, \alpha) := \Delta(\psi_F, \alpha)_3$

$\delta_1(\psi_F, \alpha_M) = 0.171\,mm$

$\delta_2(\psi_F, \alpha_M) = 0.081\,mm$

$\theta_3(\psi_F, \alpha_M) = -1.214\,deg$

Cas particuliers

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Définition Atan.mcd(R)

Quart de cercle

$\psi_{AB} := 90 \cdot deg$

$L := R \cdot \psi_{AB}$ $\quad L = 32.987\,mm$

$$S_{90} := \frac{1}{E \cdot I_{33}} \cdot \begin{bmatrix} \left(\frac{3\cdot\pi}{4} - 2\right)\cdot R^3 & \frac{1}{2}\cdot R^3 & \left(1 - \frac{\pi}{2}\right)\cdot R^2 \cdot m \\ \frac{1}{2}\cdot R^3 & \frac{\pi}{4}\cdot R^3 & -R^2 \cdot m \\ \left(1 - \frac{\pi}{2}\right)\cdot R^2 \cdot m & -R^2 \cdot m & \frac{\pi}{2}\cdot R \cdot m^2 \end{bmatrix} \cdot \frac{N}{m}$$

$\psi_F := \psi_{AB}$

$\delta_1(\psi_F, \psi_{AB}) = 0.486\,mm$ $\quad \delta_2(\psi_F, \psi_{AB}) = 0.63\,mm$ $\quad \theta_3(\psi_F, \psi_{AB}) = -1.441\,deg$

Graphe de la déformation

$n := 201$ $\quad i := 1 .. n$ $\quad \alpha_i := \frac{\psi_{AB}}{n-1}\cdot(i-1)$ $\quad x_0(\alpha) := R\cdot\cos(\alpha)$ $\quad y_0(\alpha) := R\cdot\sin(\alpha)$ $\quad r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

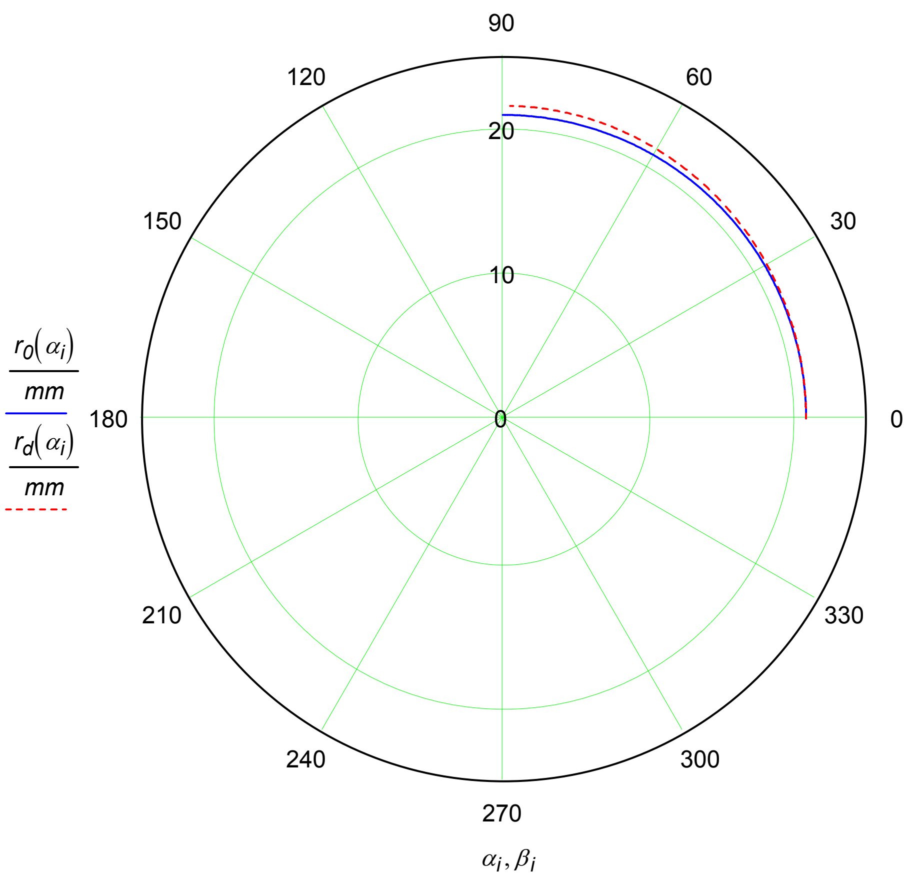
$x_d(\alpha) := x_0(\alpha) + \delta_1(\psi_F, \alpha)$ $\quad y_d(\alpha) := y_0(\alpha) + \delta_2(\psi_F, \alpha)$ $\quad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i))$ $\quad \beta_1 = 0\,deg$ $\quad \beta_n = 88.714\,deg$

$x'_d(\alpha) := \frac{d}{d\alpha}x_d(\alpha)$ $\quad y'_d(\alpha) := \frac{d}{d\alpha}y_d(\alpha)$ $\quad \alpha_0 := 0$ $\quad \alpha_{max} := \psi_{AB}$

$$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$$

$L_d = 32.999\,mm$ $\quad L = 32.987\,mm$



Demi-cercle

$\psi_{AB} := 180 \cdot deg$

$L := R \cdot \psi_{AB}$ $\quad L = 65.973\, mm$

$\psi_F := \psi_{AB}$

$$S_{180} := \frac{1}{E \cdot I_{33}} \cdot \begin{pmatrix} \frac{\pi}{2} \cdot R^3 & -2 \cdot R^3 & 2 \cdot R^2 \cdot m \\ -2 \cdot R^3 & \frac{3 \cdot \pi}{2} \cdot R^3 & -\pi \cdot R^2 \cdot m \\ 2 \cdot R^2 \cdot m & -\pi \cdot R^2 \cdot m & \pi \cdot R \cdot m^2 \end{pmatrix} \cdot \frac{N}{m}$$

$\delta_1(\psi_F, \psi_{AB}) = 0.945\, mm$ $\quad \delta_2(\psi_F, \psi_{AB}) = 0.614\, mm$ $\quad \theta_3(\psi_F, \psi_{AB}) = 2.736\, deg$

Graphe de la déformation

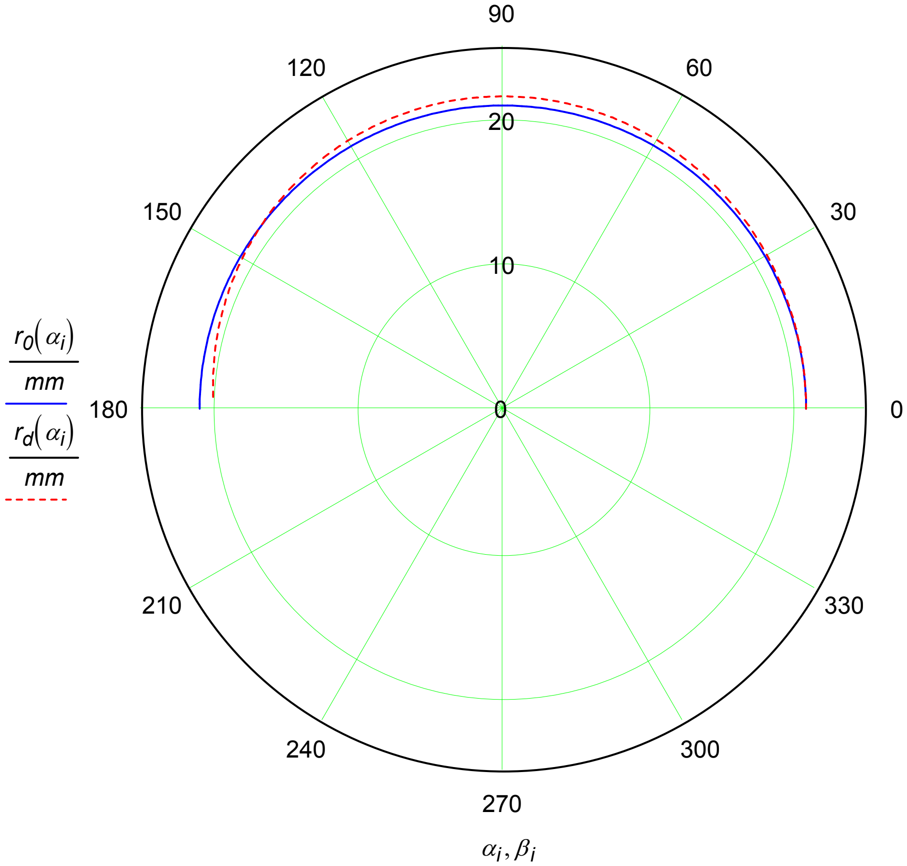
$n := 201$ $\quad i := 1 .. n$ $\quad \alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i-1)$ $\quad x_0(\alpha) := R \cdot \cos(\alpha)$ $\quad y_0(\alpha) := R \cdot \sin(\alpha)$ $\quad r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

$x_d(\alpha) := x_0(\alpha) + \delta_1(\psi_F, \alpha)$ $\quad y_d(\alpha) := y_0(\alpha) + \delta_2(\psi_F, \alpha)$ $\quad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i))$ $\quad \beta_1 = 0\, deg$ $\quad \beta_n = 178.246\, deg$

$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha)$ $\quad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha)$ $\quad \alpha_0 := 0$ $\quad \alpha_{max} := \psi_{AB}$

$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$ $\quad L_d = 65.996\, mm$ $\quad L = 65.973\, mm$



Anneau fendu

$\psi_{AB} := 360 \cdot deg$

$L := R \cdot \psi_{AB}$ $\quad L = 131.947\,mm$

$$S_{360} := \frac{1}{E \cdot I_{33}} \cdot \begin{pmatrix} \pi \cdot R^3 & 0 & 0 \\ 0 & 3 \cdot \pi \cdot R^3 & 2 \cdot \pi \cdot R^2 \cdot m \\ 0 & 2 \cdot \pi \cdot R^2 \cdot m & 2\pi \cdot R \cdot m^2 \end{pmatrix} \cdot \frac{N}{m}$$

$\psi_F := \psi_{AB}$

$\delta_1(\psi_F, \psi_{AB}) = 3.234\,mm$ $\quad \delta_2(\psi_F, \psi_{AB}) = 14.057\,mm$ $\quad \theta_3(\psi_F, \psi_{AB}) = 29.53\,deg$

Graphe de la déformation

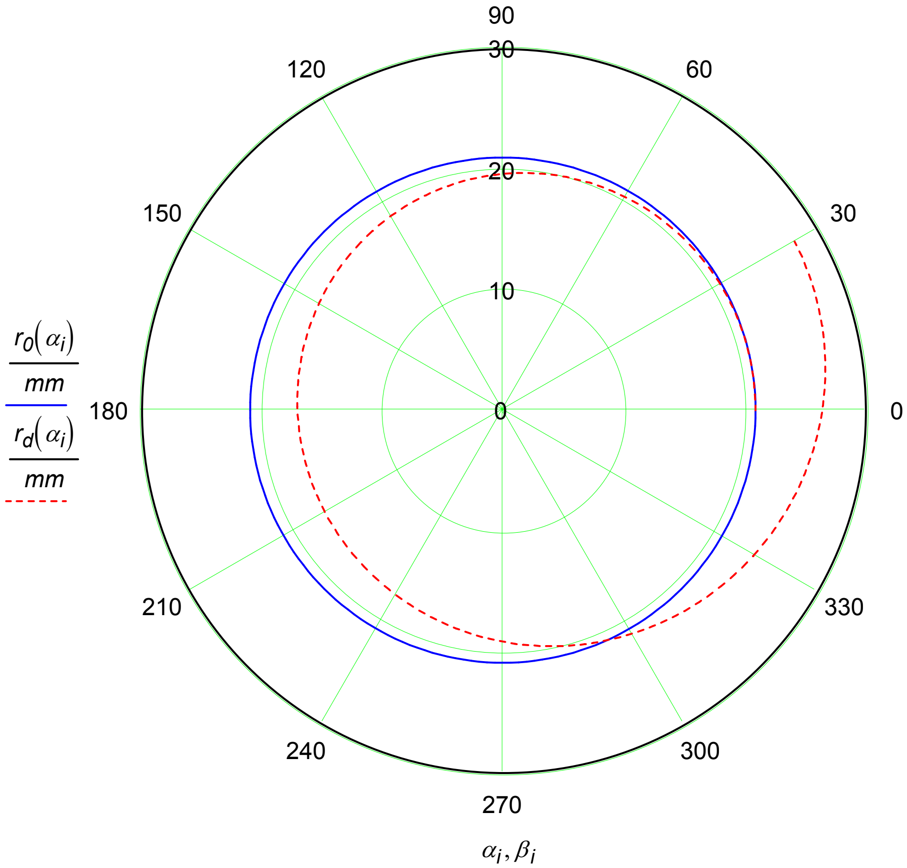
$n := 201$ $\quad i := 1..n$ $\quad \alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i-1)$ $\quad x_0(\alpha) := R \cdot cos(\alpha)$ $\quad y_0(\alpha) := R \cdot sin(\alpha)$ $\quad r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

$x_d(\alpha) := x_0(\alpha) + \delta_1(\psi_F, \alpha)$ $\quad y_d(\alpha) := y_0(\alpha) + \delta_2(\psi_F, \alpha)$ $\quad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i))$ $\quad \beta_1 = 0\,deg$ $\quad \beta_n = 30.116\,deg$

$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha)$ $\quad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha)$ $\quad \alpha_0 := 0$ $\quad \alpha_{max} := \psi_{AB}$

$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$ $\quad L_d = 139.914\,mm$ $\quad L = 131.947\,mm$



Anneau fendu avec forces extérieures appliquées en ψ = 180°
Déplacement rigide du second demi-cercle non contraint

$\psi_0 := 180 \cdot deg$ $\quad \psi_1 := 360 \cdot deg$ $\quad \Delta\psi := \psi_1 - \psi_0$ $\quad L := R \cdot \psi_1$ $\quad L = 131.947\,mm$

$\delta_1(\psi_0, \psi_0) = 0.945\,mm$ $\quad \delta_2(\psi_0, \psi_0) = 0.614\,mm$ $\quad \theta_3(\psi_0, \psi_0) = 2.736\,deg$

Déformation élastique du premier demi-cercle

$n := 401 \quad i := 1..n \quad \alpha_i := \frac{\psi_1}{n-1}\cdot(i-1) \quad x_0(\alpha) := R\cdot cos(\alpha) \quad y_0(\alpha) := R\cdot sin(\alpha)$

$x_{déf}(\alpha) := x_0(\alpha) + \delta_1(\psi_0, \alpha) \quad y_{déf}(\alpha) := y_0(\alpha) + \delta_2(\psi_0, \alpha)$

Déplacement rigide du second demi-cercle

$x_t := \delta_1(\psi_0, \psi_0) \quad y_t := \delta_2(\psi_0, \psi_0) \quad \theta := \theta_3(\psi_0, \psi_0)$

$\mathbf{O}(\psi_0) := \begin{pmatrix} x_0(\psi_0) \\ y_0(\psi_0) \end{pmatrix} \quad \mathbf{R}(\theta) := \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix} \quad \mathbf{T} := \begin{pmatrix} x_t \\ y_t \end{pmatrix}$

$\mathbf{X_d}(\alpha) := \mathbf{R}(\theta)\cdot\left[\begin{pmatrix} x_0(\alpha) \\ y_0(\alpha) \end{pmatrix} - \mathbf{O}(\psi_0)\right] + \mathbf{O}(\psi_0) + \mathbf{T} \quad x_{dr}(\alpha) := \mathbf{X_d}(\alpha)_1 \quad y_{dr}(\alpha) := \mathbf{X_d}(\alpha)_2$

$x_d(\alpha) := x_{déf}(\alpha)\cdot(\alpha \le \psi_0) + x_{dr}(\alpha)\cdot(\alpha > \psi_0) \quad y_d(\alpha) := y_{déf}(\alpha)\cdot(\alpha \le \psi_0) + y_{dr}(\alpha)\cdot(\alpha > \psi_0)$

$x_d(\psi_1) - x_0(\psi_1) = 0.897\,mm \quad y_d(\psi_1) - y_0(\psi_1) = 2.619\,mm$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i)) \quad \beta_1 = 0\,deg \quad \beta_n = 6.82\,deg$

$r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2} \quad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$x'_d(\alpha) := \frac{d}{d\alpha}x_d(\alpha) \quad y'_d(\alpha) := \frac{d}{d\alpha}y_d(\alpha) \quad \alpha_0 := 0 \quad \alpha_{max} := \psi_1$

$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\,d\alpha \quad L_d = 131.97\,mm \quad L = 131.947\,mm$